



IIT ROORKEE



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CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 53

Forward vs Futures Prices

J P Singh

Department of Management Studies



A BRIEF RECAP: KEY TAKEAWAYS SO FAR

- Arbitrage free forward price $F(0,T)=S(0)\exp(rT)$
- Forwards are private contracts, futures are exchange traded.
- Futures carry no default risk, performance guaranteed by clearing house.
- Clearing house protects itself against loss due to default by **marking to market & margining.**

- Due to MTM, maximum loss due to default restricted to one day's price change.
- Incidence of MTM default detected on very next day.
- This default risk also covered by adequate margins.

FORWARD VS FUTURES PRICES

- Consider a futures contract which lasts for T days.
- The price at the end of day t is F_t ($0 \leq t \leq T$).
- Define d as the risk-free rate of interest per day, compounded daily.
- We assume that d is constant.



- Consider the following strategy:
- At the end of day 0 (start of contract), go long $(1+d)$ contracts.
- At the end of day 1, increase long position to $(1+d)^2$ contracts.
- At the end of day 2, increase the long position to $(1+d)^3$ contracts and so on.
- At the beginning of day t , the long position will be $(1+d)^t$ contracts.
- At the end of the day this will lead to a profit (possibly negative) given by: $(F_t - F_{t-1}) (1+d)^t$

- Suppose that this profit is compounded daily at the rate d per day till the day T .
- Its value at the end of day T is: $(F_t - F_{t-1}) (1+d)^t (1+d)^{T-t} = (F_t - F_{t-1}) (1+d)^T$
- Thus at the end of day T , the entire investment will have accumulated to:

$$\sum_{t=1}^T (F_t - F_{t-1}) (1+d)^T = (1+d)^T \sum_{t=1}^T (F_t - F_{t-1})$$

- $= (F_T - F_0) (1+d)^T = (S_T - F_0) (1+d)^T$
- The last equality follows because on the day the contract matures, the futures price must equal the spot price of the underlying asset.
- The strategy above coupled with an investment of F_0 in risk-free bond at the end of day 0, will give a terminal value of: $F_0(1+d)^T + (S_T - F_0) (1+d)^T = S_T(1+d)^T$
- This combined strategy requires an investment of F_0 at the end of day 0 and no further investment.

- Now suppose a forward contract is available on the same asset.
- Let the forward price at the end of day 0 be G_0 .
- Consider the following strategy:
- Invest G_0 in a risk-free bond and buy forward contract on $(1+d)^T$ units of underlying.
- At the end of day T , the investment will have matured to $G_0 (1+d)^T$.
- This is used to pay for the delivery on the forward contract.
- The investor has an asset worth $S_T (1+d)^T$.



- Thus the combined risk-free bond and futures strategy and the risk-free bond plus forward strategy have identical terminal payoffs.
- To prevent arbitrage, the initial investment requirements must also be identical. Hence $F_0 = G_0$ i.e. futures and forward prices are identical.

IMPORTANT OBSERVATIONS

- No arbitrage considerations create a direct relationship between forward and spot prices e.g. $F(0,T)=S(0)*e^{rT}$.
- No arbitrage considerations also mandate convergence between forward and futures prices.
- Thus, futures prices should also move in tandem with the spot prices.

- **Nevertheless, since futures are freely tradeable, their prices, ultimately (at the macro level), are determined by the interaction of demand and supply at the marketplace.**
- **However, the demand and supply, themselves, are functions of fundamental variables.**

- These fundamental variables also manifest themselves as determinants of the inputs that go into the intrinsic value e.g. the cash flow projections, its risk profile etc.
- The net result is that futures prices hover in close vicinity of forward prices although occasional divergence would subsist due to heterogeneous risk-return preferences, market asymmetries and anomalies.
- But the divergences cannot be unreasonably large.